Chiral Symmetry Restoration and Eigenvalue Density of Dirac Operator at Finite Temperature

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1. Introduction

Flavor-Chiral Symmetries of QCD at low T

$$U(N_f)_L \otimes U(N_f)_R \Rightarrow U(1)_B \otimes S(N_f)_L \otimes SU(N_f)_R \Rightarrow U(1)_B \otimes S(N_f)_V$$

flavor

chiral anomaly
(explicit breaking)

spontaneous breaking of
chiral symmetry

QCD at high T

deconfinement (QGP)

restoration of chiral symmetry

phase transition

low T $U(1)_B \otimes S(N_f)_V$ $\Rightarrow$ high T $U(1)_B \otimes S(N_f)_L \otimes SU(N_f)_R$

Questions in this study

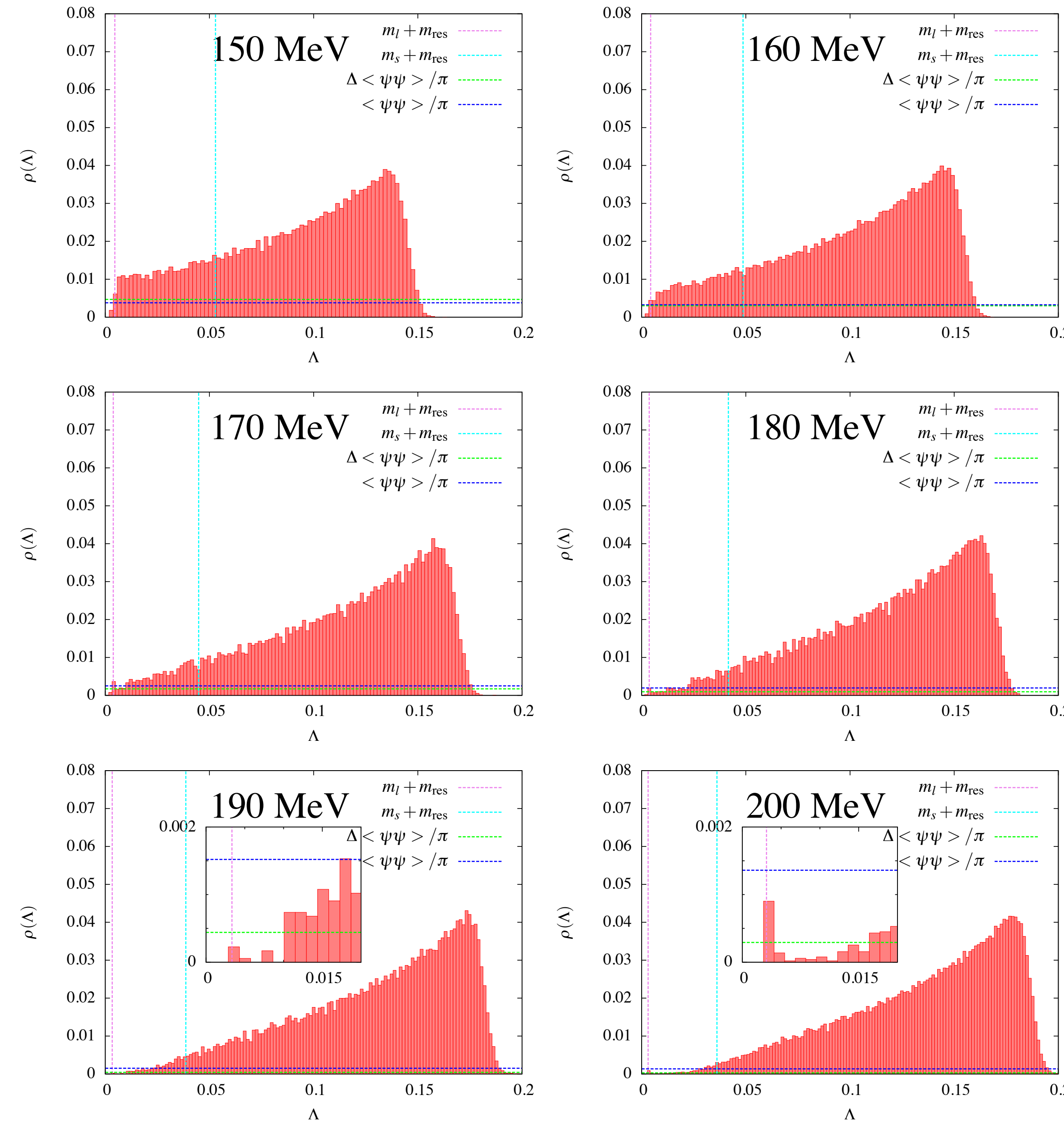
1. Constraints to eigenvalue density: $$\rho(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$$

2. Constraint to singlet susceptibility: $$\chi_{\rm singlet} = \int d^4x \; [\pi(x)\pi(0) - \eta(x)\eta(0)]$$

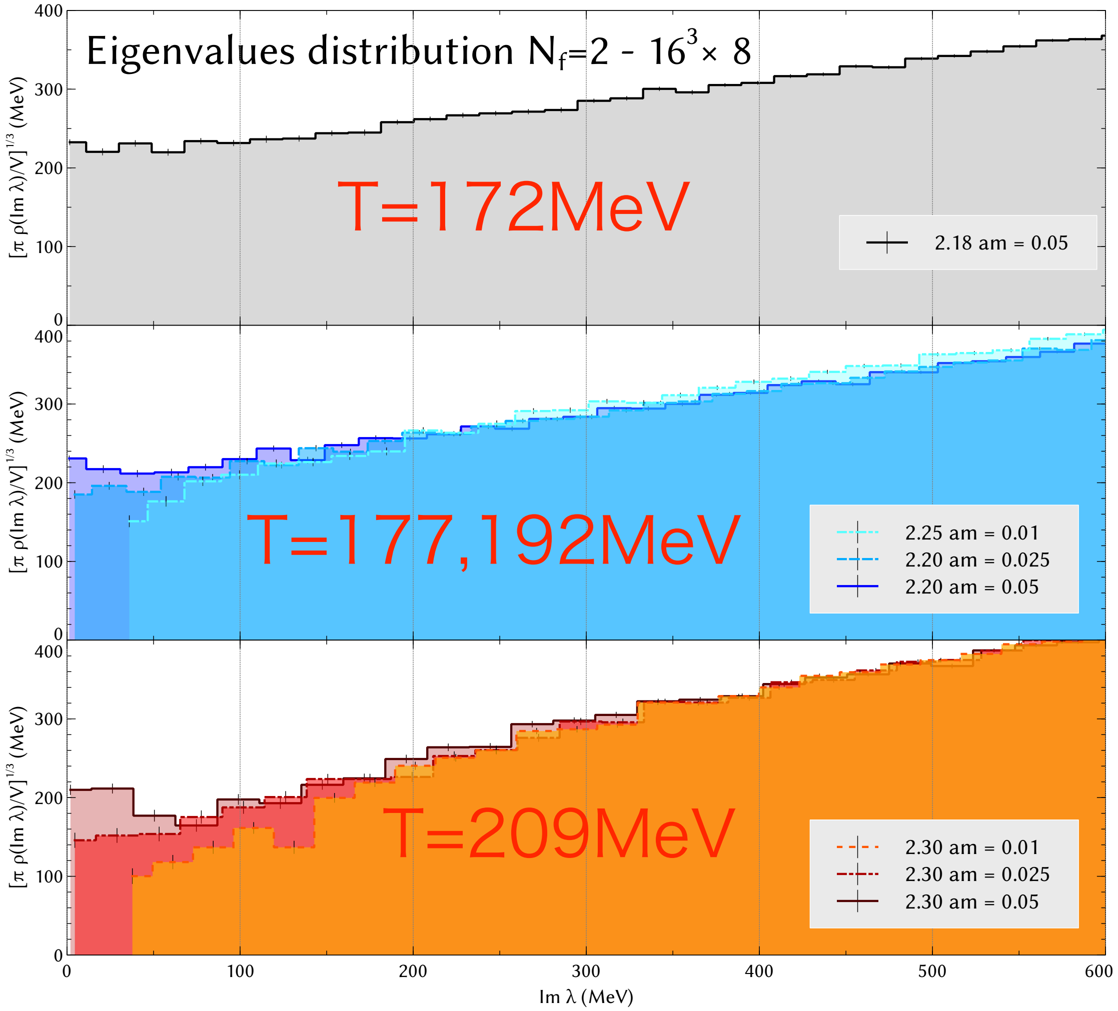
Previous studies

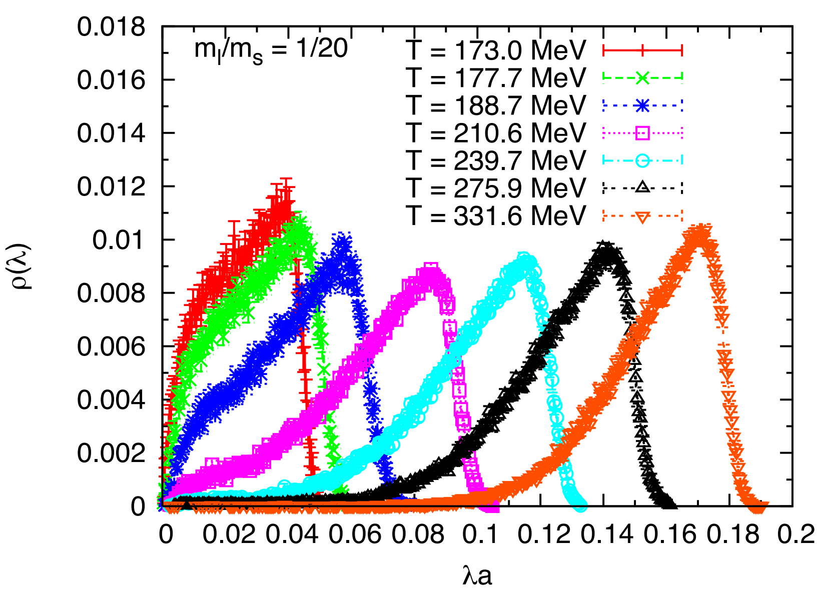
$$\rho(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$$

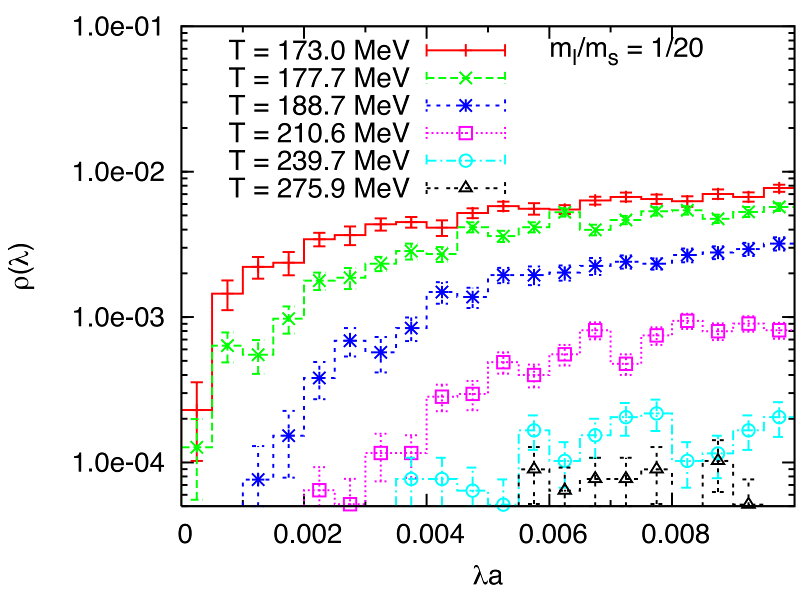
Lin (HotQCD11), DW



Cossu *et al.* (JLQCD11), Overlap



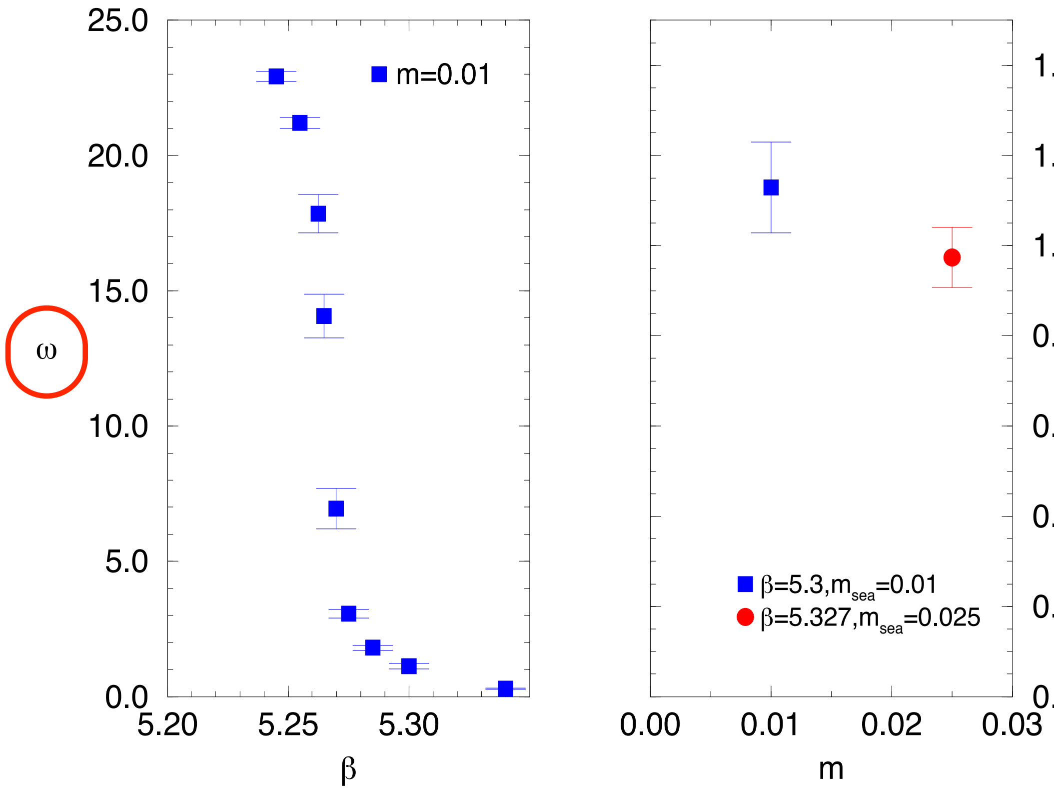




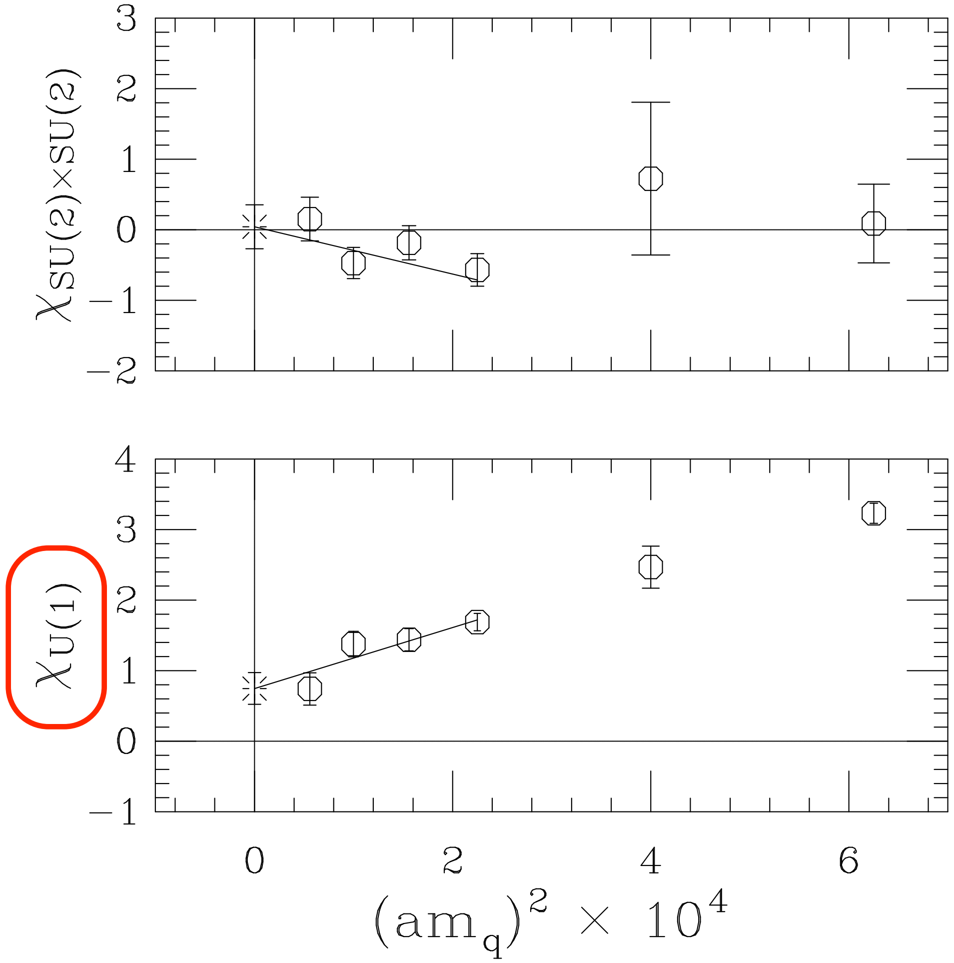
Ohno *et al.* (11), HISQ

Is a gap open at $\lambda = 0$?

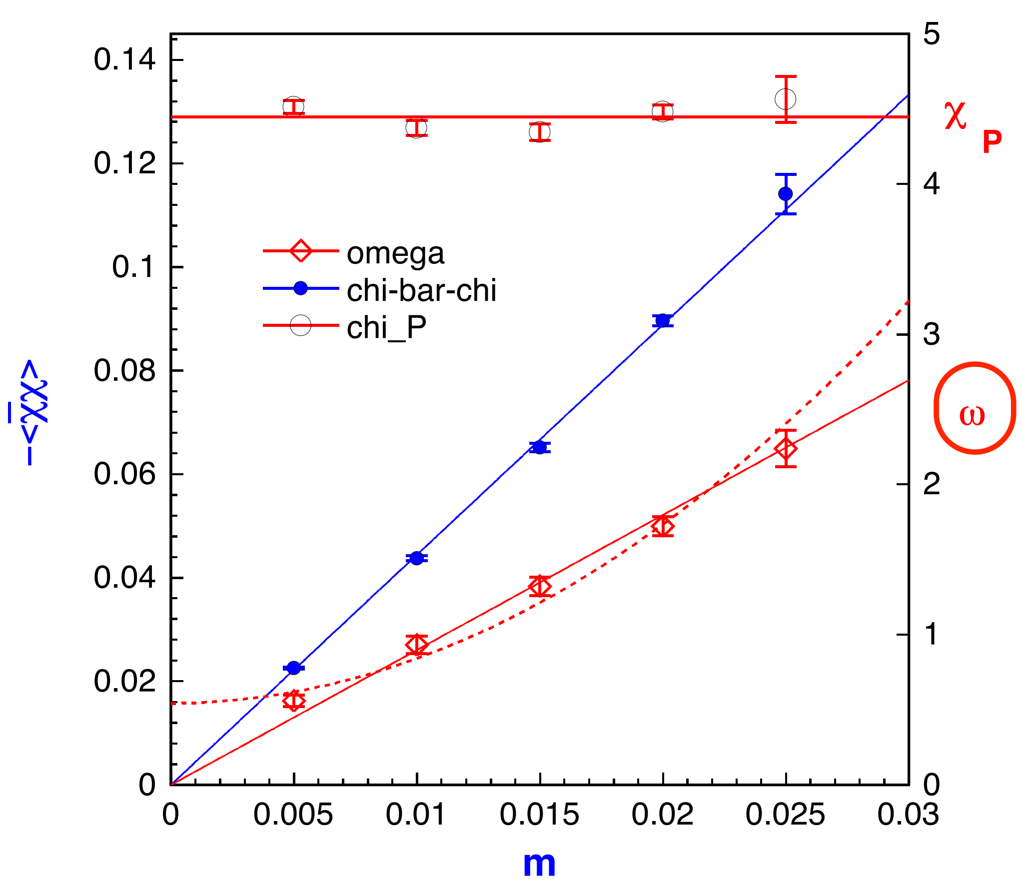
Chandrasekharan-Christ(95), KS



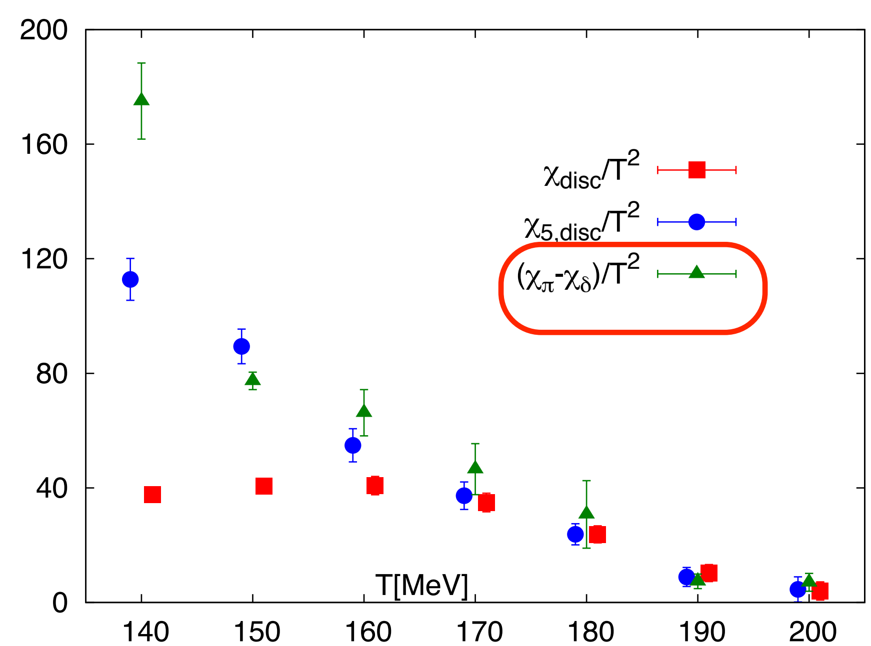
Bernald, *et al.* (96), KS



Chandrasekharan *et al.*, (98), KS



Hegde (HotQCD11), DW



$$\chi_{\text{singlet}} = 0 \ ?$$

This talk

give constraints to eigenvalue densities of 2-flavor overlap fermions, if chiral symmetry in QCD is restored at finite temperature.
discuss a behavior of singlet susceptibility using the constraints.

Content

1. Introduction
2. Overlap fermions
3. Constraints to eigenvalue densities
4. Discussions: singlet susceptibility

2. Overlap fermions

Action $S = \bar{\psi}[D - mF(D)]\psi, \quad F(D) = 1 - \frac{Ra}{2} D$

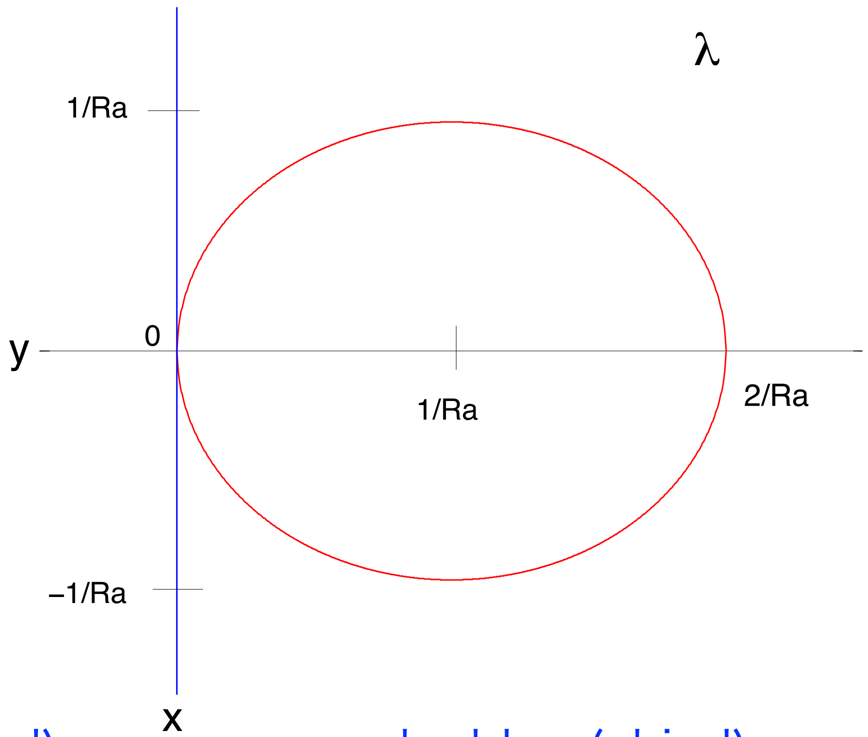
Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$

Eigenvalue spectrum

$$\lambda_n^A + \bar{\lambda}_n^A = aR\bar{\lambda}_n^A \lambda_n^A$$

$$D(A)\phi_n^A = \lambda_n^A \phi_n^A$$

$$\Rightarrow \quad D(A)\gamma_5\phi_n^A = \bar{\lambda}_n^A \gamma_5 \phi_n^A$$



Propagator

bulk modes(non-chiral) zero modes(chiral) doublers(chiral)

$$S(x,y) = \sum_n \left[\frac{\phi_n(x)\phi_n^\dagger(y)}{f_m\lambda_n - m} + \frac{\gamma_5\phi_n(x)\phi_n^\dagger(y)\gamma_5}{f_m\bar{\lambda}_n - m} \right] - \sum_{k=1}^{N_{R+L}} \frac{1}{m}\phi_k(x)\phi_k^\dagger(y) + \sum_{K=1}^{N_D} \frac{Ra}{2}\phi_K(x)\phi_K^\dagger(y)$$

$$f_m = 1 + \frac{Rma}{2}$$

Measure $P_m(A) = e^{-S_{YM}(A)}(-m)^{N_f N_{R+L}^A} \left(\frac{2}{Ra}\right)^{N_f N_D^A} \prod_{\Im\lambda_n^A > 0} (Z_m^2 \bar{\lambda}_n^A \lambda_n^A + m^2)$

$$Z_m^2 = 1 - (ma)^2 \frac{R^2}{4}$$

positive definite and even function of $m \neq 0$ for even N_f

Ward-Takahashi identities under “chiral” rotation

$$\theta^a(x)\delta_x^a\psi(x) = i\theta^a(x)T^a\gamma_5(1-RaD)\psi(x),$$
$$\theta^a(x)\delta_x^a\bar\psi(x) = i\bar\psi(x)\theta^a(x)T^a\gamma_5,$$

$$\langle (J_x^a - \delta_x^a S)O + \delta_x^a\mathcal{O}\rangle = 0$$

$$\int d^4x \langle\{J_x^a + 2mP^a(x)\}\mathcal{O} + \delta_x^a\mathcal{O}\rangle = 0.$$

integrated WTI

“measure” term (anomaly)

$$J_x^a = -2i\mathrm{tr}\, T^a\gamma_5\left(1-\frac{R}{2}aD\right)(x,x) = -\delta^{a0}2iN_f\mathrm{tr}\,\gamma_5\left(1-\frac{R}{2}aD\right)(x,x)$$

Operators

$$S^a(x) = \bar\psi(x)T^aF(D)\psi(x),$$ scalar

$$P^a(x) = \bar\psi(x)T^ai\gamma_5F(D)\psi(x),$$ pseudo-scalar

$$S^a = \int d^4x\, S^a(x), \quad P^a = \int d^4x\, P^a(x)$$

chiral rotation at N_f=2

$$\delta^bS^a = 2\delta^{ab}P^a, \quad \delta^bP^a = -2\delta^{ab}S^a$$
$$\delta^0S^a = \delta^aS^0 = 2P^a, \quad \delta^0P^a = \delta^aP^0 = -2S^a$$

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4} \qquad N = \sum_i n_i, \quad n_1+n_2 = \mathrm{odd}, \ n_1+n_3 = \mathrm{odd}$$

$$\boxed{\lim_{m\to 0}\langle\delta^a\mathcal{O}_{n_1,n_2,n_3,n_4}\rangle_m = 0}$$

if the chiral symmetry is restored.

$$\frac{\delta^a}{2}\mathcal{O}_{n_1,n_2,n_3,n_4} = -n_1\mathcal{O}_{n_1-1,n_2,n_3,n_4+1} + n_2\mathcal{O}_{n_1,n_2-1,n_3+1,n_4} - n_3\mathcal{O}_{n_1,n_2+1,n_3-1,n_4} + n_4\mathcal{O}_{n_1+1,n_2,n_3,n_4-1}$$

3. Constraints to eigenvalue densities

N=1 $\quad \mathcal{O}_{1,0,0,0} = P^a, \ \delta^a \mathcal{O}_{1,0,0,0} = -2S^0$

$$\frac{1}{V}\langle S^0 \rangle_m = \underbrace{\frac{N_f}{mV}\langle N^A_{R+L}\rangle_m}_{\to 0 \text{ as } V \to \infty} + N_f \langle I_1 \rangle_m$$

$$\langle \mathcal{O}^A \rangle_m = \frac{1}{Z}\int \mathcal{D}A \, P_m(A) \mathcal{O}^A$$

$$I_1 \to \frac{1}{Z_m}\int_0^{\Lambda_R} d\lambda \, \underline{\rho^A(\lambda)} \frac{2m_R}{\lambda^2 + m_R^2} g_0(\lambda)$$

$$m_R = \frac{m}{Z_m}, \ g(\lambda) = 1 - \frac{\lambda^2}{\Lambda_R^2}$$

$\Lambda_R = \frac{2}{Ra}$: cut-off

$$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V}\sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{k=0}^{\infty} \rho_k^A \frac{|\lambda|^k}{k!} = \rho_0^A + \rho_1^A|\lambda| + \cdots,$$

$$\Rightarrow \quad I_1 = \pi \rho_0^A + O(m)$$

$$\lim_{m\to 0}\lim_{V\to\infty}\frac{1}{V}\langle S^0 \rangle_m = 0 \quad \Rightarrow \quad \lim_{m\to 0}\langle \rho_0^A \rangle_m = 0$$

$$\Rightarrow \quad \langle \rho_0^A \rangle_m = O(m^2)$$

1st constraint

$\boxed{\text{N=2}}$

$$\chi^{\sigma-\pi} = \frac{1}{V^2}\langle S_0^2 - P_a^2\rangle_m, \qquad \chi^{\eta-\delta} = \frac{1}{V}\langle P_0^2 - S_a^2\rangle_m$$

$$\chi^{\sigma-\pi} = \left\langle N_f^2\left(\frac{1}{mV}N_{R+L} + I_1\right)^2\right\rangle_m + O\left(\frac{1}{V}\right)$$

$$\Rightarrow \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{m^2V^2}\underbrace{\langle(N_{R+L}^A)^2\rangle_m}_{O(m^{N_fV})} = \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{mV}\underbrace{\langle N_{R+L}^A\,\rho_0^A\rangle_m}_{O(m^{N_fV})} = \lim_{m\to 0}\underbrace{\langle(\rho_0^A)^2\rangle_m}_{O(m^2)} = 0$$

These conditions are automatically satisfied.

$$\chi^{\eta-\delta} = N_f\left\langle\frac{1}{m^2V}\{2N_{R+L} - N_fQ(A)^2\} + \underline{\frac{1}{Z_m}\left(\frac{I_1}{m_R} + I_2\right)}\right\rangle_m \qquad Q(A) = N_R^A - N_L^A$$

$$\frac{I_1}{m_R} + I_2 = \rho_0^A\left(\frac{\pi_m}{m} + \frac{2}{\Lambda_R}\right) + 2\rho_1^A + O(m), \qquad I_2 = \frac{2}{Z_m}\int_0^{\Lambda_R}d\lambda\,\rho^A(\lambda)\,\frac{m_R^2 - \lambda^2 g_0(\lambda^2)g_m}{(\lambda^2+m_R^2)^2}, \quad g_m = \frac{1}{Z_m^2}\left(1 + \frac{m^2}{2\Lambda_R^2}\right)$$

$$\lim_{m\to 0}\chi^{\eta-\delta} = 0 \Rightarrow \lim_{m\to 0}\underbrace{\frac{N_f^2\langle Q(A)^2\rangle_m}{m^2V}}_{O(m^{N_f\sqrt{V}-2})} = 2\lim_{m\to 0}\langle\rho_1^A\rangle_m$$

$$\Rightarrow \langle\rho_1^A\rangle_m = O(m^2)$$

2nd constraint

N=3

$$\langle \mathcal{O}_{2001}\rangle_m \to 0, \quad \langle -\mathcal{O}_{0201}+2\mathcal{O}_{1110}\rangle_m \to 0, \quad \langle \mathcal{O}_{0021}+2\mathcal{O}_{1110}\rangle_m = 0$$

$$\langle -\mathcal{O}_{0003}+2\mathcal{O}_{2001}\rangle_m \to 0, \quad \langle \mathcal{O}_{0021}-\mathcal{O}_{0201}+\mathcal{O}_{1110}\rangle_m \to 0,$$

$$\lim_{m\to 0}\lim_{V\to\infty}\frac{\chi_3}{V^4} = \pi N_f \lim_{m\to 0}\left[\frac{\langle \rho_0^A\rangle_m}{m^2} + \underbrace{\frac{2\bar{\rho}_1}{m\pi}}_{O(m)} + \frac{\langle \rho_2^A\rangle_m}{2}\right], = 0$$

+ positivity

$$\langle \rho_0^A\rangle_m = O(m^4),\ \langle \rho_2^A\rangle_m = O(m^2)$$

3rd constraints

N=4

No additional constraints

Final results

$$\lim_{m\to 0}\langle \rho^A(\lambda)\rangle_m = \lim_{m\to 0}\langle \rho_3^A\rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

4. Discussion:Singlet susceptbility

Singlet susceptibility

$$\chi^{\pi-\eta} = \lim_{V\to\infty} \frac{N_f^2}{m^2 V} \langle Q(A)^2 \rangle_m = (m^{N_f \sqrt{V}-2})$$

$$\lim_{m\to} \chi^{\pi-\eta} = 0$$

singlet susceptibility becomes zero if the chiral symmetry is recovered at hight T.

This, however, does not mean U(1)_A symmetry is recovered at high T.

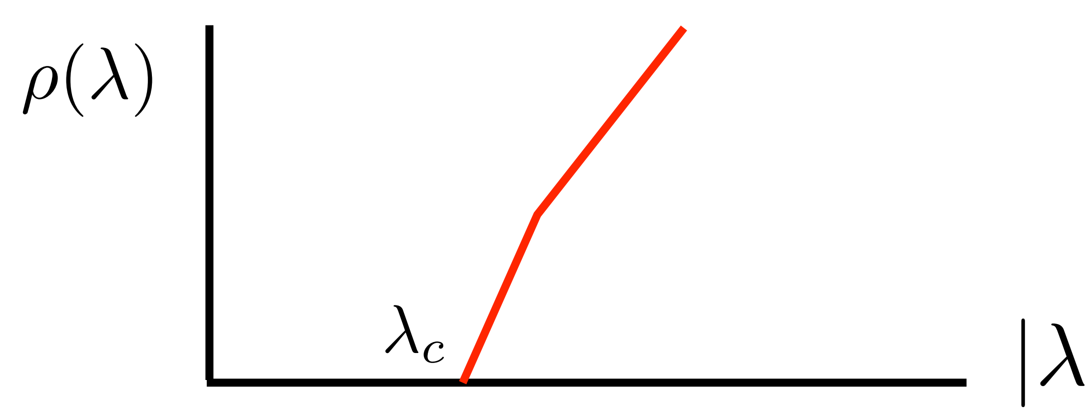
$\lim_{m\to} \chi^{\pi-\eta} = 0$ is necessary but NOT sufficient for the recovery of U(1)_A .

Future

new constraints at N > 4 ?

eigenvalues of Dirac operator

have a gap near zero ?



$$\rho(\lambda) = 0 \text{ at } |\lambda| \leq \lambda_c$$